MATH 1326 Reference Guide

Differentiation

Differentiation Rules

Linearity

$$\frac{d}{dx}\left[u+v\right] = u' + v'$$

$$\frac{d}{dx}\left[cu\right] = cu'$$

Product & Quotient Rules

$$\frac{d}{dx}\left[uv\right] = u'v + v'u$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - v'u}{v^2}$$

Chain Rule

$$\frac{d}{dx}\left[f(u)\right] = f'(u) \cdot u'$$

Derivative Identities

$$\frac{d}{dx}\left[c\right] = 0 \qquad \frac{d}{dx}\left[x\right] = 1 \qquad \frac{d}{dx}\left[x^n\right] = nx^{n-1} \qquad \frac{d}{dx}\left[e^x\right] = e^x \qquad \frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[u\right] = u' \qquad \frac{d}{dx}\left[u^n\right] = nu^{n-1} \cdot u' \qquad \frac{d}{dx}\left[e^u\right] = e^u \cdot u' \qquad \frac{d}{dx}\left[\ln u\right] = \frac{u'}{u}$$

Integration

Integration Rules

Linearity

$$\int \left[f(x) + g(x)\right] dx = \int f(x)\, dx + \int g(x)\, dx$$

$$\int af(x)\, dx = a \int f(x)\, dx$$

Integration by Parts

$$\int u\, dv = uv - \int v\, du$$

Integral Identities

$$\int 0\, dx = C \qquad \int dx = x + C$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x}\, dx = \ln|x| + C \qquad \int e^x\, dx = e^x + C$$

$$\int (ax+b)^n\, dx = \frac{1}{a} \cdot \frac{(ax+b)^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{ax+b}\, dx = \frac{1}{a} \cdot \ln|ax+b| + C \qquad \int e^{ax+b}\, dx = \frac{1}{a} \cdot e^{ax+b} + C$$

Fundamental Theorem of Calculus

$$F'(x) = f(x) \implies \int_a^b f(x)\, dx = F(b) - F(a)$$

Functions of Two Variables

2nd Derivative Test

Compute $D = (f_{xx})(f_{yy}) - (f_{xy})^2$.
Let (x_0, y_0) be a critical point.

- If $D(x_0, y_0) < 0$, then (x_0, y_0) is a saddle point.
- If $D(x_0, y_0) > 0$, then...
  - if $f_{xx}(x_0, y_0) > 0$, then (x_0, y_0) is a local minimum.
  - if $f_{xx}(x_0, y_0) < 0$, then (x_0, y_0) is a local maximum.

Lagrange Multipliers

Objective function: $f(x, y)$
Constraint: $g(x, y) = 0$

$$L(x, y, \lambda) = f(x, y) - \lambda g(x, y)$$

1. Compute L_x, L_y, and set them equal to 0.
2. Solve for λ in both $L_x = 0$ and $L_y = 0$.
3. Set the two expressions for λ equal to each other, and solve for x or y.
4. Substitute into the constraint equation.

Total Differential

$$dz = f_x(x, y)dx + f_y(x, y)dy$$

$$f(x + dx, y + dy) \approx f(x, y) + dz$$

Linear Differential Equations

$$y' + P(x)y = Q(x)$$

Integrating Factor: $I(x) = e^{\int P(x)\, dx}$

Solution: $y = \dfrac{1}{I(x)}\left[\int Q(x)I(x)\, dx + C\right]$

Geometric Sequences & Series

Notation

r = common ratio
a = first term
a_n = n^{th} term
S_n = sum of first n terms

Formulas

$$a_n = ar^{n-1}$$

$$S_n = \frac{a[1 - r^{n+1}]}{1 - r}$$

Ordinary Annuities

Notation

R = payment per period
i = interest per period
n = number of periods
P = present value
S = total value
y = remaining value after x periods

Formulas

$$S = \frac{R[(1 + i)^n - 1]}{i}$$

$$P = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$y = \frac{R[1 - (1 + i)^{-(n-x)}]}{i}$$

Source: Lial, Greenwell, Ritchey (2016). *Calculus with Applications, 11e*. Pearson.